NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from March Number.]

Corollary II. But again I am able hence to show that those two straights AX, BX, meeting which the straight $PFHD$ makes either two internal angles toward the same parts equal to two right angles, or consequently (from Eu. I. 13 and 15) alternate external or internal angles equal to one another, or again, from the same cause, an external (as suppose DHX) equal to an internal and opposite HFX; that, say I, those two straights not even in their infinite production can meet one another.

For if from any point N of AX is let fall to BX the perpendicular NR, this will be in the hypothesis of acute angle (which alone in any case can hinder us) greater (from III. Cor. I.) than the common perpendicular KL. Therefore those two straights AX, BX cannot ever meet one another.

But furthermore here thou hast demonstrated propositions 27 and 28 of the first book of Euclid, and indeed without immediate dependence from the preceding 16 and 17 of the same first book, about which difficulties could arise when the triangle should be of infinite sides on a finite base; to which sort of a triangle without doubt would refer one who believed that these two straights AX, BX met one another at least at an infinite distance, although the angles at the transversal $PFHD$ were such as we have supposed.

Moreover, on account of the demonstrated common perpendicular KL, surely those two KX, LX cannot come together toward the part of the points X, since also (from a superposition easily understood) toward the other part also would meet at the same time the remaining and themselves unterminated KA, LB. Wherefore two straights AX, BX would enclose a space; which is contrary to the nature of the straight line.

But these things are later. For in the preceding I have never applied either the 16th or 17th of the first book of Euclid, except where clearly it treats of a triangle bounded on every side, as indeed I promised I would so take care to do *in Proemio ad Lectorem.*

[To be Continued.]